Toward a Functional Accounting System: Accounting Variances and Statistical Variance Analysis

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The functions of accounting and statistics are in most respects similar. These fields are both tools, concerned with the processes of measurement, collection of data, analysis and decision making. In performing their functions both accounting and statistics rely heavily on signals generated by variations from expected performance.

Even in their manifested differences these two fields are at worst complementary. For example, accounting is concerned with the design of the total system which will provide the relevant information on a continuous basis, while statistics focuses more on *ad hoc* micro analyses such as design and analysis of experiments for eliminating hypotheses, which are aimed at determining "relevance". In view of all the similarities and complementarities, one wonders why statistical analyses have not become an integral part of the accounting systems.

The purpose of this paper is to point out some ways by means of which statistical techniques can be introduced into the accounting system to

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make its output more meaningful and provide flexibility for measuring the potential outcome of alternative courses of action. To do this, I will:

(a) Provide some background material on my previous efforts in this direction and also elaborate a little more on what motivated me in the first place, (b) show that the output of standard accounting systems can be used for statistical analyses of variance, and (c) carry out some of the consequences of statistical analyses and of testing hypotheses for what I call functional managerial accounting systems.

In an appendix, I will apply such statistical techniques to determine the statistical significance of variations in the performance of subentities within the firm, using as inputs the variances generated by the accounting system. Hopefully the arguments to be presented in this paper and the illustration will encourage the designers of managerial information systems to make such analyses an integral part of the accounting systems.

I. Background and Motivation

On another occasion[1] I sketched the general characteristics of an integrated accounting system of the future. The criteria that I set before me in developing such a system were twofold. I wanted first of all to provide management with routine quantitative information, useful in analyzing the efficacy of existing organizational configurations, and second I felt that such information should be a part of a comprehensive managerial accounting system. It was an attempt, in other words, to integrate all the major requirements for information that is necessary for managerial decision making, present such information on a routine rather than *ad hoc* basis, and so

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[1]Zenon S. Zannetos, "Measuring the Efficiency of Organization Structures: Some Implications for the Control System of the Firm" Working Paper 117-65, Alfred P. Sloan School of Management, MIT, 1965, pp. 19-21.

help management encompass in its decisions more of the global aspects of the firm's activities. And this because I strongly believe that as far as management is concerned:

1. The efficiency of allocation of resources given the objectives, cannot be divorced from the process of defining the objectives themselves, and,

2. An operationally meaningful definition of objectives and the design of the organizational structure of the firm are virtually two sides of the same coin.

Present-day managerial accounting systems are mostly concerned with the short-run process of resource allocation but even this in a rather restrictive sense. The standard cost system in industrial operations is the only branch of routine accounting which attempts to provide through variances, information on how efficiently resources are utilized. But as I have previously pointed out, "Unlike statistical variance analysis which attaches probabilistic interpretation to the results obtained, accounting variances do not indicate what is important and what is not."[2] Furthermore, one has no way of obtaining routine cause and effect relationships, or information on the process of defining and translating objectives, which process is inter-

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[2] Zenon S. Zannetos, "On the Mathematics of Variance Analysis", The Accounting Review, Volume XXXVIII, No. 3, July 1963, p. 530; For a possible solution to this problem see, __________, "Standard Costs as a First Step to Probabilistic Control: A Theoretical Justification, an Extension and Implications", The Accounting Review, Vol. XXXIX, No. 2, April 1964, pp. 296-304.

woven with the issues of centralization and decentralization.[3] Unless we make some progress in these directions, we cannot very well help management with our measurements.

In an effort to provide answers to some of the problems that plague accounting, and also make the information generated more useful for managerial decisions, I suggested: (a) A method of covariance analysis for assessing the efficiency of existing organization structures, and (b) statistical analysis of variance (the components of covariances) for developing elementary cause and effect relationships to be incorporated in the information system of the firm.[4] Now I wish to illustrate how one can use the data generated by the traditional "standard cost" system as inputs to a managerial accounting system, where the latter is based on cause and effect relationships derived from statistical variance analysis. These relationships are necessary to get a functional accounting system started and then change it sequentially as the evidence dictates.

II. Accounting Variances and Statistical Variance Analysis: Theoretical Formulations

Let us now start with a brief statement of the mathematical basis of statsitical variance analysis--using as an example a single-factor completely

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[3]For the factors affecting the organizational structure of the firm and the role played by the definition of objectives in this process see: Zenon S. Zannetos, "On the Theory of Divisional Structures: Some Aspects of Centralization and Decentralization of Control and Decision Making", Management Science, December 1965, Vol. 12, No. 4, Series B, pp. 49-69. Also see Herbert A. Simon, "On the Concept of Organizational Goal", Administrative Science Quarterly, Vol. 9, No. 1, June 1964, pp. 1-22.

[4]"Measuring the Efficiency of Organization Structures ...etc.," Op. Cit.

randomized experiment with random treatment levels--and prove that accounting variances out of "standard" systems can serve as input data. Those familiar with the subject at hand, will readily recognize that the proof is very simple, and that it falls out immediately upon making two critical equivalence substitutions, (a) and (b) as stated below, in the test statistic.

Let: G = the budgeted goal or the control standard

X_{ij} = observation i of subsample j

$X_{ij}-G$ = the accounting variance V_{ij} generated by observation i of subsample j

$V._j$ = the total accounting variance generated within subsample j (subsidiary account)

$V..$ = the grand total accounting variance (summary account) generated by all observations for all j

$\bar{X}..$ = the oberall average performance as observed

n_j = the number of observations or entries in subsample j

N = the grand total number of observations

Clearly, <u>if we deal with accounting variances rather than observations the test statistics remains unchanged</u>. And this because the transformation results in the subtraction of a constant, that is to say G, from each and every observation. Consequently we substitute in the F statistic:

(a) $\bar{X}.. = V../N$

(b) $\bar{X}._j = V._j/n_j$

and the proof is complete.

For those now who are not well versed in the mathematics of statistical analysis of variance more explanation is in order. So we go back to the "fundamental equation of one-way analysis of variance":

$$(1) \quad \sum_{j=1}^{k} \sum_{i=1}^{n_j} (X_{ij}-\bar{X}_{..})^2 = \sum_{j=1}^{k} n_j (\bar{X}_{.j}-\bar{X}_{..})^2 + \sum_{j=1}^{k} \sum_{i=1}^{n_j} (X_{ij}-\bar{X}_{.j})^2$$

The above relationship tells us that the total sum of squared deviations around the grand mean is equal to the sum of the squares of the deviations between treatment means and the grand mean, plus the sum of the squares of the error or the sum of squared deviations within treatments.

If we divide the terms on the right-hand side of expression (1) by their respective degrees of freedom, then we obtain two independent estimates of the variance σ^2 when H_o is true, that is to say when the only difference between the observations within the treatments (subsamples) is an estimate of the error variance σ^2_e . These estimates are chi-square distributed and their ratio is F distributed with k-1 and N-k degrees of freedom. So the test statistic is:

$$(2) \qquad F_{k-1,\ N-k} = \frac{\sum_{j=1}^{k} n_j (\bar{X}_{.j}-X_{..})^2/k-1}{\sum_{j=1}^{k} \sum_{i=1}^{n_j} (X_{ij}-\bar{X}_{.j})^2/N-k}$$

Note now that in the numerator we have:

$$(3) \quad \sum_{j=1}^{k} n_j (\bar{X}._j - \bar{X}..)^2 = \sum_{j=1}^{k} n_j \bar{X}^2._j - 2 \sum_{j=1}^{k} n_j \bar{X}._j \bar{X}.. + N\bar{X}^2..$$

We proceed with substitutions (a) and (b) to obtain:

$$(4) \quad = \sum_{j=1}^{k} n_j V^2._j / n^2_j - 2 \sum_{j=1}^{k} n_j (V._j / n_j)(V../N) + NV^2../N^2$$

$$(5) \quad = \sum_{j=1}^{k} V^2._j / n_j - 2 V^2../N + V^2../N$$

$$(6) \quad = \sum_{j=1}^{k} V^2._j / n_j - V^2../N \qquad \text{with } k-1 \text{ df}$$

Therefore, all the necessary data for the numerator of the F ratio wherever the latter is applicable, can thus be obtained from the output of the regular accounting process.

In a similar fashion, the sum of squares of the denominator can be also expressed in terms of accounting variances as follows:

$$(7) \quad \sum_{j=1}^{k} \sum_{i=1}^{n_j} (X_{ij}-\bar{X}_{\cdot j})^2 = \sum_{j=1}^{k} \sum_{i=1}^{n_j} V^2{}_{ij} - 2 \sum_{j=1}^{k} V^2{}_{\cdot j}/n_j + \sum_{j=1}^{k} V^2{}_{\cdot j}/n_j$$

$$(8) \quad = \sum_{j=1}^{k} \sum_{i=1}^{n_j} V^2{}_{ij} - \sum_{j=1}^{k} V^2{}_{\cdot j}/n_j \quad \text{with } N\text{-}k \text{ df.}$$

The above completes our proof that the necessary data inputs for statistical variance analysis, where such analysis is applicable, can be obtained from the subsidiary and control or summary accounts under standard accounting systems.

Obviously statistical analysis of variance cannot be reasonably performed wherever a statistical universe cannot be properly defined and its parameters cannot be estimated. Similarily, accounting variances are meaningless, for performance evaluation and learning, in the absence of standardizable operations or at least a process of rough estimation of the relevant parameters. Consequently, the preconditions for statistical and accounting variance analysis are the same and any distinction between the two is for the most part artificial. The "standard cost" system of accounting, which is based on requirements similar to those needed for statistical analysis, has been in operation for years. Its existence and successful use show that, at least in the area of manufacturing operations, the field is ripe for the introduction of statistical techniques for the purpose of generating more useful managerial information. But there are many more operations, both of manufacturing and non-manufacturing nature, where these approaches can be extended, because methods for defining meaningful universes and estimating

means (standards) and variances do exist.[5] Our efforts here are aimed at developing the techniques which will bridge the gap between the statistics and accounting and thus speed up the process of assimilation of the former into the latter.

The illustration, which is included in the appendix, proceeds in a step by step fashion to apply aspects of such statistical analysis techniques to the output of the accounting process. The appendix is not aimed at the expert in statistics nor at the manager who possesses no statistical knowledge at all, but at the vast majority of people in between, who are interested and wish to acquire working knowledge in the general area of information for managerial planning and control. The main body of the paper, however, was structured to stand on its own without any study of the appendix.

III. Cause and Effect Relationships as Inputs to Managerial Accounting Systems

The analysis suggested in this paper indicates that the variances generated by the accounting system should be: (a) automatically tested for probabilistic significance for focusing attention on "exceptions" that require managerial action, and (b) used as inputs to experimental designs for statistical

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[5] Although for certain statistical tests for determining the probabilistic significance of accounting variances the standard must be set at the mean, small errors in estimation are not incapacitating especially if the standards are tested *a posteriori* in a Bayesian framework. For a brief description of the latter method see: Zenon S. Zannetos, "Mathematics as a Tool of Accounting Instruction and Research", *The Accounting Review*, Vol. XXXVIII, No. 2, April 1963, pp. 326-335 and "Standard Costs as a First Step to Probabilistic Control", *op. cit.* The possibility of using biased standards for purposes of motivation does not interfere with this analysis, in fact it makes it more necessary as well as revealing.

variance analysis. The latter will provide certain cause and effect relationships that are presently not part of the managerial accounting systems. The value of these relationships cannot be overemphasized, and runs in several directions:

1. Performance Evaluation

The evaluation of performance is not only a multi-dimensional but also a multi-varied process itself. It may be aimed at (a) measuring the results of experimentation and discerning changes in technology where issues of motivation are, or are assumed to be absent, (b) encouraging learning through experimentation in identifying cause and effect in cases where proper motivation is assumed to exist, and (c) motivating efficient behavior by challenging the purpose of individual actions through the feedback-control mechanism of the management information system.

(a) Discerning changes in Technology: In order to be able to assess the impact of technological change on the results from operations, one must first separate the impact of random variations surrounding a given state of technology from the consequences of purposive action by operating units. Since operations take place within a probabilistic setting, a certain amount of random variation is to be expected. As knowledge is gained, however, and specialized information is generated about a particular technological stage, this type of random variation is reduced. An efficient managerial information system should, therefore, separate the random from non-random variations in the performance of otherwise homogeneous units within standardized operations. If the differences between the average performance of operating subentities (departments, etc.) are great, then some type of technological change must be taking place, on which management should capitalize. Present accounting systems tend to aim at

"deterministic conformity" and thus suppress the informational content of meaningful signals that emanate from suboperations. Our previous suggestions and the system proposed in this paper tend to remedy these deficiencies.

(b) Encouraging Learning through Experimentation: Performance evaluation, of course, is not an end in itself nor is it primarily aimed at rewarding efficiency, although the latter is one of the legitimate objectives of any evaluation process. If nothing else, people look at reward as a signal which reinforces certain modes of behavior. It is in effect part of the individual information system of associating cause and effect. Another important aspect of an efficient system of performance evaluation, however, is that it encourages experimentation. If the tools for measurement and the establishment of cause and effect relationships exist, then managers will be more inclined to experiment for learning, because they will not be moving in the dark. Furthermore, with such a system, information will flow in "continuously" to allow sequential learning and adaptation, and thus minimize the probability of disastrous results. For if experimentation has to be carried out inflexibly to its completion before any evaluation is performed and information on cause and effect obtained, disasters will occur often, and will thus discourage long-run and substantial experiments.[6]

(c) Motivating Efficient Behavior: An information system which is based on functional relationships can be geared to carry out the consequences of projected actions (on the basis of postulated cause and effect relationships) and provide managers with bases for choosing among alternatives. The separation of the various components of variance, and the closer identification of the direct impact of managerial decisions on global objectives, will

[6] It is for these reasons that a standard system can encourage innovation if used properly, because it provides a substability or a base for experimentation at the margin without endangering the total objective.

help direct management attention in areas consistent with the overall objectives of the firm. This is especially important at middle management levels where the operational objectives are means to an "obscure" end, and often appear to be in conflict with the personal goals of the decision maker. Signals will be generated to warn of impending changes, and point out the necessity for future action because of variations that originated in other units within the firm or for that matter outside the firm. On the basis of these signals both position and performance budgets will be revised automatically to incorporate in them the latest information.

2. System Efficiency

As operations increase in size and complexity, the requirements imposed upon the information system of the firm increase exponentially. I have previously shown, in conjunction with centralization and decentralization, that the channels of communication needed for linking the members of structures organized for mutual interaction, increase by more than c^2, where c stands for the increase in the size of the group.[7] In terms of information storage requirements the situation is even worse. If we take as an example the data inputs to our present budgetary planning and control systems, the total number of possible combinations of data for extraction of information and hence the total number of possible pieces of data generated for storage at various levels in the hierarchy is 2^n, where

[7] See "On the Theory of Divisional Structures ... etc.," op. cit.

n is the size of the raw data pool.[8] Of course, not under all circumstances will we have to take all possible combinations, but with the present practices of aggregation and storage of semi-processed information (pooled data), the data stored will not be much less. Furthermore, the present combinations of raw data do not usually result in real information useful for managerial decisions, because the transformation functions (the functional relationships for cause and effect) are not given. Consequently, memoranda or information storage devices outside the regular accounting system are necessary for control and decision making thus further adding to the requirements for storage. One example of this is the information necessary for planning which is mostly carried on a memorandum basis.

My suggestions for a functional accounting system, which as I have previously sketched operates on a raw data base and functional forms,[9] will alleviate this situation because it will obviate the need for transmitting as well as storing so much redundancy.

The efficiency of an information system undoubtedly depends on both the quantity of useful raw data upon which it draws, and the intelligence or manipulative capability of the system. Given a certain capacity in a system, the more of it we use for storage purposes the less capacity we have for

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[8] We can readily see that the total N is:

$$N = \sum_{x=0}^{n} B(x,n) = \sum_{x=0}^{n} B(x,n)(1)^{x}(1)^{n-x} = (1+1)^{n}$$

where $B(x,n)$ stands for binomial coefficient of n items combined x at a time.

[9] See "Measuring the Efficiency of Organization Structures...etc.," op. cit.

transformations. Furthermore, the probability of chaos, cluttering, and confusion increases with the amount of data stored. As a result it appears that the greater the sophistication of the information system the more emphasis should be placed on its manipulative capabilities. This we also observe in human beings. There are people who have stored in their brain an extraordinary amount of data which they can readily retrieve. However, the data are usually disjoint and rather useless for the owner (with the possible exception of impressing others at social gatherings), because the frame of reference is missing. Intelligence depends extensively on manipulative capability. A person who has to depend on memory must store the information in practically all the various semi-processed forms he will need to use later, while the one who depends on his intelligence stores only primitive data and methods of analysis (cause and effect relationships), leaving the particular need, whenever manifested, to dictate the transformation which will result in the best possible information for the particular use. The former system may be faster but inflexible and limited, the latter a bit slower but more articulate, fundamental and useful. The majority of existing accounting systems are of the former type, unsophisticated and useful mostly for storage of disjoint classifications of data. Our suggestions for eliminating semi-processed information and storing functional forms is aimed at providing "intelligence" to the management information system.

3. Advance Warning of Changes in Interrelated Operations

One of the greatest attributes of an information system is its prognostic capacity. The longer the time span between the prediction of the consequences of certain events and the point of occurrence of such, if no adjusting actions are undertaken, the greater the value of the information. An information

system based on cause and effect relationships can progressively advance from the most obvious or immediate relationships to the most fundamental, thus increasing continuously the lead time available for action.

Many of the operations of business firms are vitally interrelated and can be easily expressed in terms of cause and effect relationships. Such an arrangement will not only aid in performance valuation, but will also provide better premises for operating decisions. The cause and effect relations will guide the information system as well as the managers, in selecting the type of information that must be transmitted and how often it must be made available. Today, managers are only guided by the most superficial cause and effect relationships in collecting data for decisions, and are often neglecting the global aspects of the firm's operations since the organizational structure shields them from the interrelationships between their limited goals and the overall objectives of the firm. A functional information system will not only provide the necessary data upon request, but will also bring them to the attention of management without the latter's initiation, if significant changes in other operations necessitate action by a certain unit. It will allow "decentralized" operations to function independently more effectively, and at the same time permit overall management to use the generated substabilities for higher level solutions. In effect the use of the organization structure of the firm as a management tool will be exploited fully and be brought to fruition, while subunits realize their full potential under the greatest possible decentralization. And this because such arrangements

will provide operating units with advance signals of the necessity of impending actions, and thus allow them to plan and implement changes with a minimum of delay and wastage of resources.

4. Development of a Total Modular Model of Operations

Clearly, a functional accounting system is based on a model of the firm's operations. The budgetary system is itself one such model. In recent years increasingly more and more efforts have been devoted to the development of different types of total models.[10] Some of the latter are based on simulated relationships of existing or assumed systems, and others on simulation of observed or postulated decision-making processes. Because of the complexity of the firm's total operations, analytical and normative approaches are effectively limited to only parts of the total system.

Most of the models that have been thus far proposed are either *ad hoc*, or independent of the system which will generate the necessary data for their continued application. In my estimation the models cannot be divorced from the information system of the firm. If such empirical independence is observed it implies that either the model is addressed toward inconsequential objectives, or else the information system is misguided in collecting useless information.

My efforts, as reflected in this as well as previous papers, are aimed toward remedying the existing deficiencies in managerial accounting systems.

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[10] For some such efforts as related to the total information systems of the firm see: Jay W. Forrester, *Industrial Dynamics*, The MIT Press, Cambridge, Massachusetts, 1961; Charles P. Bonini, *Simulation of Information and Decision Systems in the Firm*, Prentice-Hall, Inc., Englewood Cliffs, New Jersey, 1963; James C. Emery, "The Planning Process and Its Formalization in Computer Models", Sloan School of Management, MIT, Working Paper No. 108-65, 1965.

The models that I suggest we introduce in the information systems derive both from theoretical functional relations--the experimental design--and also directly from the operations in that the results from operations are used in testing alternative hypotheses. Thus normative as well as behavioral notions may be both brought to bear on the design of the information system of an organization. The cause and effect relationships that are developed are used locally for purposes of learning and efficient resource allocation, but at the same time they serve as modules in an integrated information system. Thus, the planner and the operating manager, in the presence of such a system, can test the consequences of proposed actions and be able to encompass more of the global criteria in their decisions. Finally, we have in the proposed system--through covariance and variance analysis--a means for system validation on a continuous basis, because the validation is part of the system itself. One of the most severe criticisms of simulated models of the firm and of simulation information systems, has been the absence of statistical validation and the necessity for accepting or rejecting them mostly on faith alone. A system based on the suggestions presented in this paper will satisfy at least partly a lot of the necessary requirements for a successful statistical validation of the models.

5. Eventual Development of an Associative Information System

In a functional accounting system there is one-to-one correspondence between inputs (the variables) and the functional forms into which these enter. This implies that with the exception of the subjective prior distributions attached to the levels of the various functional relationships, the model is monolithic. In other words it does not allow for different configurations or qualitative differences in the postulated functional relationships, and

therefore cannot automatically generate and show the implications of many alternative plans for the utilization of given inputs. These deficiencies we cannot today remedy by introducing the desirable attributes into our information systems, because we have not made enough progress in understanding the process by means of which objectives are translated into operations.[11] It is imperative, however, that we solve this problem, because only then we will be able to introduce meaningful economic opportunity costs into our information and control systems.

The successor to the functional accounting system will be probably an associative information system, which associates inputs with functional forms in a one-many relationship. It will draw data from the same data base as used by a functional accounting system. A reference file of variables both dependent and independent will provide a multi-dimensional cross reference of the variables and the various forms into which these enter as inputs, as well as the conditional patterns which determine the applicability of the forms. Depending on the availability-scarcity of complementary resources, the system will then associate the input with a configuration of functional relationships, suggest a plan of utilization (based on the current opportunity costs of resources), and compare it with the budgeted dominant solution. The manager will thus be able to choose the best feasible plan of action of else query the system for further information before a decision is made.

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[11] This topic I will attempt to analyze in a forthcoming paper entitled, "Objectives and Transformations: Toward a Theory of Dominance".

The <u>functional</u> accounting system is feasible with present day ancillary technology and knowledge. The <u>associative</u> information system, however, is still out in the future in that there are a lot of problems, both conceptual and technical, that need be resolved before the system becomes a reality. On purely theoretical grounds, however, the signs are very encouraging, so it may not be too long before a breakthrough in this area is achieved. Until then, our life will not be dull because we will find an abundance of challenges in trying to implement a functional managerial accounting system.

IV. Brief Summary

I have suggested in this paper (and shown in the appendix) that accounting data, especially of the managerial type, are not as inflexible or useless as many claim. Most of the present criticism is applicable to, and therefore should be directed at, the faulty use of information, and not at the validity of the basic data, pointing out that the greatest payoff lies in the improvement of the managerial decision-making processes. Refinements of data--if such refinements are addressed to the end uses--are helpful, but the fact still remains that we have not as yet utilized or capitalized on the information content of existing data. In particular I have shown previously and here, that data generated by existing standard accounting systems can be extremely useful for managerial purposes, if extended, serving as inputs to probabilistic control systems, design and analysis of experiments, and accounting systems based on cause and effect relationships. The implications of all these potential uses are in our estimation enormous. They can

bring to bear on managerial decisions, both the normative and behavioral aspects of operations, modeling and statistical validation, and the full extent of utilization of all the available tools and ancillary technologies. The end result will be an accounting system which is really managerial.

APPENDIX

The Use of Accounting Variances for Statistical Variance Analysis: An Illustration

1. The Analysis of Variance

Let us assume that we have a company which produces a standard line of products. Each product is manufactured by many divisions (or departments, groups, machines, etc. within a division), under standard batch-order conditions.[12] The material (or labor) usage variances for the division are very small, indicating that "operations are under control" since the standards are set by this company at the expected value or mean.[13] We want to find out, however, whether there are differences between the performance of the various departments, operators, machines, weeks, etc.

In the example that we use here for a one-way analysis of variance, we shall test at the departmental level, by picking at random four departments and using the last five batches completed by each as the basis for testing efficiency in terms of materials usage.[14] Then we shall compare Department 1 versus Department 3, Department 2 versus Department 4 and the average performance of Departments 1 and 3 versus that of Departments 2 and 4. And

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[12] Although I will be talking about batch-order manufacturing operations, the arguments and system are equally applicable to any standardizable operations. Also, a lot of the assumptions made here are not limiting in any fundamental sense but are introduced to describe the situation I use for illustrative purposes.

[13] That is to say the company applies initially a test to assess the probabilistic significance of total deviations similar to the one described in "Standard Costs as a First Step to Probabilistic Control" *op. cit.*

[14] The analysis of variance can be applied as a matter of routine to all the results of each accounting period, rather than on a sampling basis. Such a practice, however, will be wasteful.

this because we wish to discern possible particularities in technology, etc., that create heterogeneities in otherwise standard operations, and which heterogeneities are of such magnitude as to imply the possible existence of subpopulations. Such analysis is necessary for learning purposes and for incorporating innovations in standard operation procedures.

In our example we assume that the estimated material content (standard) for each homogeneous batch is 1,000 units, and that the observations which we pick out of the "credit side" of the Work-in-Process departmental accounts are as shown in Table 1.

Table 1

	Departments			
Batches	1	2	3	4
1	1004	1012	994	993
2	998	1015	995	996
3	1012	1007	986	994
4	1008	1000	997	990
5	1010	1005	996	992

On the basis of our assumptions that the material content of the standard batch output is 1,000 material units, the Material Usage Variance accounts of the various departments will provide the information V_{ij} shown in the upper part of Table 2. Note here that "debit" balances are designated with minus signs.

Table 2

Material Usage Variances V_{ij}

by Departments per Batch

	1	2	3	4	
V_{ij}:	4	12	-6	-7	
	-2	15	-5	-4	
	12	7	-14	-6	
	8	0	-3	-10	
	10	5	-4	-8	
$\sum_{i=1}^{n_j} V_{ij} = V_{\cdot j}$:	32	39	-32	-35	$\sum_{j=1}^{k} V_{\cdot j} = V_{\cdot\cdot} = 4$
n_j:	5	5	5	5	$\sum_{j=1}^{k} n_j = N = 20$
$\bar{V}_{\cdot j}$:	6.4	7.8	-6.4	-7	$\bar{V}_{\cdot\cdot} = .2$
$\sum_{i=1}^{n_j} V^2_{ij}$:	328	443	282	265	$\sum_{j=1}^{k} \sum_{i=1}^{n_j} V^2_{ij} = 1318$

In Table 2:

V_{ij} = the material usage variance of batch i for department j

$V_{\cdot j}$ = the total material usage variance for department j for the batches used in the sample. If the check is done randomly on a monthly basis then $V_{\cdot j}$ represents the monthly total departmental variance.

n_j = the number of batches taken for department j. Note that n_j need not be the same for each department.

N = $\sum_{j=1}^{k} n_j$ or the total number of batches included in the experiment.

$V_{\cdot\cdot}$ = the total material usage variance for the division based on the observations included in the sample. If the firm is committed to 100 per cent sampling--a practice that we find unnecessary--then $V_{\cdot\cdot}$ represents the total variance summary or control account.

$\bar{V}_{\cdot j}$ = the average observed variance per batch for department j. The combined sum of $\bar{V}_{\cdot j}$ and the standard output, can serve as an estimate of the mean departmental performance to be used in a Bayesian framework.

$\bar{V}_{\cdot\cdot}$ = the overall average observed variance per batch. The combined total of $\bar{V}_{\cdot\cdot}$ and the standard can serve as an estimate of the empirical grand mean output for the division. Again here this is useful for Bayesian analysis.

As shown in the text in part II, we can test the hypothesis that the departmental variances σ^2_D are equal to zero, by comparing the observed F value at k-1 and N-k degrees of freedom (df) with the critical region of the F distribution.[15]

This empirical F value is:

$$F_{k-1,N-k} = \frac{\left(\sum_{j=1}^{k} V^2_{.j}/n_j - V^2_{..}/N\right)/k-1}{\left(\sum_{j=1}^{k} \sum_{i=1}^{n_j} V^2_{ij} - \sum_{j=1}^{k} V^2_{.j}/n_j\right)/N-k}$$

where in the numerator we have, in our case, the estimate of the variance between departments, and in the denominator the estimate of the variance within departments or the random error ε_{ij}. The amounts in the parentheses are the sum of squares (SS) and the total of the numerator and denominator is, of course, the sum of squares of the decomposed variance,

$$SS_T = \sum_{j=1}^{k} \sum_{i=1}^{n_j} V^2_{ij} - V^2_{..}/N$$

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[15]The critical region is usually the upper tail of the F distribution, and the test consists of rejecting the hypothesis H_o if the observed F ratio value is greater or equal to the value of $F_{1-\alpha}$, where α is the confidence level at which we wish to test.

The values of the above terms as derived from the data presented in Table 2 are for our illustration as follows:

1. Total sum of Squares (N-1 df.) $= \sum_{j=1}^{k} \sum_{i=1}^{n_j} V_{ij} - V^2../N$

$$= 1318 - \frac{16}{20}$$

$$= 1317.2$$

2. Sum of Squares <u>between</u> Departments

$$SS_D \text{ (k-1 df.)} = \sum_{j=1}^{k} V^2._j/n_j - V^2../N$$

$$= \frac{1024 + 1521 + 1024 + 1225}{5} - \frac{16}{20}$$

$$= 958$$

3. Sum of Squares of Error = 1317.2 - 958 = 359.2

Now we summarize in Table 3 our results as preparatory to testing the null hypothesis H_o that the variance σ^2_D around the performance of the various departments is zero.

Table 3

Source of Variation	Degrees of Freedom	Sum of Squares	Mean Square	Expected Mean Square
Between Departments	(k-1) = 3	958	319.3	$\sigma^2_e + 5\sigma^2_D$
Within Departments or Error ε_{ij}	(N-k) = 16	359.2	22.45	σ^2_e
Total	19	1317.2		

The test statistic for the above (the F-ratio for 3 degrees of freedom for the numerator and 16 for the denominator) is:

$$F_{3,16} = 319.3/22.45 = 14.22$$

which is highly significant (the critical $F_{3,16}$ value for $\alpha = .01$ being 5.29) indicating that the hypothesis that $\sigma^2_D = 0$ must be rejected. So we can safely infer that the difference between the performance (material utilization) of the various departments contains something more than the estimate of the population variance. Probably there is a real difference in the performance of the various departments and further probing is required. A manager would naturally be interested in finding out among other things:

(a) What percentage of the total variation in material utilization among departments is due to random causes (stochastic error) and what due to the efficiency of departments or possible qualitative differences in the raw material used.

(b) Which of the average performances are significantly different? For example is Department 1 statistically more efficient than say No. 3? Is Department 2 better than 4? How does the average performance of Departments 1 and 3 compare with that of Departments 2 and 4?

(c) Can we utilize the information generated by statistical analysis for developing cause and effect information useful for budgeting purposes, performance evaluation, and more efficient utilization of resources in the case of interdependent operations?

2. The Components of Variance

The random model that we have been testing was

$$X_{ij} = \mu + D_j + \varepsilon_{ij}$$

where X_{ij} is the standard material content of the actual output of each input batch, μ = 1000 material units (the standard material content of the standard output) and ε_{ij} the random error. In terms of the material usage variance, the model was changed to:

$$(X_{ij} - \mu) = (\mu_{.j} - \mu) + (X_{ij} - \mu_{.j}) \qquad \text{or}$$

$$V_{ij} \equiv \bar{V}_{.j} + (V_{ij} - \bar{V}_{.j})$$

Setting now the observed variances equal to their expected values, we can solve for the best estimates of the components of the variance from standard, and so separate that which is due to stochastic error from the part which can be safely attributed to the average performance of departments. Thus the variance due to the latter is equal to:

$$S^2_D = (S^2_{V_{ij}} - S^2_e)/n$$

where $S^2_{V_{ij}}$ is the observed variance for batches, and S^2_e the error variance.

From Table 3 we obtain:

$$S^2_{V_{ij}} = 319.3 \text{ , and}$$

$$S^2_e = 22.45$$

Hence: $5\ S^2_D = 319.3 - 22.45$

$$S^2_D = 57.37$$

The estimate of the total variance from operations S^2_T is therefore

$$S^2_T = S^2_D + S^2_e$$

$$= 79.82$$

showing that approximately 72 per cent of the total variance can be attributed to differences between the mean performance of the departments, and 28 per cent to a random variation around the means.

If now we use the above estimate of the standard deviation $S_T = 8.9$ for assessing the probabilistic significance of variances within the division, we can set up a system as previously suggested,[16] by means of which only variances beyond the control limits will be brought to the attention of management. Also

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[16]"Standard Costs as a First Step to Probabilistic Control" <u>op. cit.</u>

this estimated standard deviation can be used for deriving the discrete probabilities applicable to (a) the various alternative standards and (b) the occurrences of various variances given the standards, under the *a priori* assumption that the standard deviation is invariant with respect to the different standards. These features of experimental design must be incorporated into the information system if we are to have selective "signal generation" and sequential testing of standards. The separation of random variations from variations due to operating efficiency and variations arising from technological change is not only desirable but it is also *requisite* for efficient decision making.

Looking over the entries in Table 2, we notice that the variations around the standard (mean) fall within $\pm 2S_T$, consequently none would have been brought to the attention of divisional management under such control limits. If alternatively the limits were set at one standard deviation (that is to say identifying as exceptional any variance which occurs due to purely stochastic reasons with probability $p < .32$) then the performance of Departments 1 and 2 would have been analyzed twice and that of Departments 3 and 4 only once during the period it took to run the specified five batches.

3. Comparisons Between Departmental Performance

To the extent that every statistical universe contains subpopulations which may have their own distinct characteristics but which characteristics become buried in averages, we must test the means of the various departments to find out which differ. We have already determined that differences do exist but we do not know exactly which means are statistically different. This contrast of means is normally used in *fixed* rather than *random*

experimental designs. That is to say, it is used under controlled experimental conditions to test whether there are any significant differences in performance because of the introduction of different methods of operation (treatment). For example one may wish to test for differences in the fastness of the color of a cloth as the temperature or the amount of time of the dyeing process is varied. Under such conditions the levels of treatment of the experiment are exhaustive.

In our case the levels are not exhaustive but are a random sample out of many. We wish, however, to contrast means in order to obtain signals on the possible existence of differences, in order that we further investigate (possibly through fixed experimental designs) and establish cause and effect relationships. Since we have decided prior to the experiment on what comparisons we wanted to make (Department 1 versus Department 3; Department 2 versus Department 4; and Departments 1 and 3 together versus Departments 2 and 4 together), we can use the method of orthogonal contrasts.[17] Again the necessary inputs for this test are obtained from the accounting records, which in our case are the Material Usage Variance accounts of the departments as shown in Table 2.

The test statistic for orthogonal contrasts is again the F distribution, where for each hypothesis, in the numerator we have the sum of squares of the contrast and in the denominator the sum of squares of the error, each (numerator and denominator) divided by its own degrees of freedom.

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[17] If the decision on comparisons is made after the data are observed, then the method of orthogonal contrasts is not appropriate but other methods are available.

A contrast C_m is defined as

$$C_m = \sum_{j=1}^{k} C_{jm} V_{.j}$$

where C_{jm} stands for the coefficient for department j in contrast C_m

and where $\sum_{j=1}^{k} n_j C_{jm} = 0$

For orthogonality the sum of the product of the coefficients of each pair of contrasts C_{jm} and C_{jq} must also be zero, so that

$$\sum_{j=1}^{k} n_j C_{jm} C_{jq} = 0$$

Finally the sum of squares of a contrast C_m is

$$SS_{C_m} = \frac{(C_m)^2}{\sum_{j=1}^{k} n_j C_{jm}^2}$$

Our three contrasts are consequently the three rows of the resulting matrix upon multiplication of the matrix of coefficients C_{jm} with the column vector of material usage variances $V_{.j}$. That is to say:

$$C_m = \begin{pmatrix} 1 & 0 & -1 & 0 \\ 0 & +1 & 0 & -1 \\ +1 & -1 & +1 & -1 \end{pmatrix} \begin{pmatrix} V._j \end{pmatrix}$$

$$C_1 = 32 + 0 - (-32) + 0 = 64$$

$$C_2 = 0 + 39 + 0 - (-35) = 74$$

$$C_3 = 32 - 39 - 32 - (-35) = -4$$

And the sum of squares is therefore:

(a) $SS_{C_1} = \frac{(64)^2}{5(2)} = 409.6$

(b) $SS_{C_2} = \frac{(74)^2}{5(2)} = 547.6$

(c) $SS_{C_3} = \frac{(-4)^2}{5(4)} = .8$

The total sum of squares, that is to say the sum of (a) through (c), is 958.0 (as before: see Table 3).

Using the above results, each of which has one degree of freedom, we test:[18]

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[18] The denominator of the F-ratio is the variance of the random error.

$H_1 : V_{\cdot 1} = V_{\cdot 3}$ $\quad F_{1,16} = \frac{409.6}{22.45} = 18.3$

$H_2 : V_{\cdot 2} = V_{\cdot 4}$ $\quad F_{1,16} = \frac{547.6}{22.45} = 24.5$

$H_3 : V_{\cdot 1} + V_{\cdot 3} = V_{\cdot 2} + V_{\cdot 4}$ $\quad F_{1,16} = \frac{.8}{22.45} = 0.036$

At the 1 per cent significance level the value of $F_{1,16} = 8.53$; consequently H_1 and H_2 are rejected but H_3 is not. We conclude, therefore, that there are significant differences between the performances of Department 1 and Department 3, also between Department 2 and Department 4, but not between the combined totals of Departments 1 and 3 and Departments 2 and 4. These results raise the possibility that Departments 1 and 2 may belong to a different statistical population than that of Departments 3 and 4. The necessity for an inquiry in the method of operation or in the quality of raw material inputs of these pairs of departments is, therefore, strongly suggested. Fixed experiments, not necessarily limited to a one-way analysis of variance, may now be run if necessary so as to isolate cause and effect. In essence a second round of analyses of variances within the standardized operations must be undertaken to isolate the real causes of variation in performance, for the purpose of learning and adopting new methods of operation. Any new method of operation being a deviation from existing standard procedures will at first serve as a challenger, become eventually the standard procedure if successful, serve as a basis (or substability) to a higher level solution, and eventually be challenged itself, *ad infinitum*.

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